Class: XII
Time Allowed: 3 Hours

Subject: Mathematics
Maximum Marks: 80

General Instructions:

1. **This Question paper contains - Four sections A, B, C and D. Each section is compulsory.**
2. **Section A-Question 1 to 10 comprises of 10 Very Very Short Answer type questions of 1 mark each**
3. **Section B-Question 11 to 20 comprises of 10 Very Short Answer (VSA)-type questions of 2 marks each.**
4. **Section C-Question 21 to 28 comprises of 8 Short Answer (SA)-type questions of 4 marks each.**
5. **Section D-Question 29 to 31 comprises of Long Answer (LA)-type questions of 6 marks each.**

SECTION A

Q.1) If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, *then the possible* value(s) of 'x' is/are

(a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

Q.2) If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$

(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

Q.3) The degree of the differential equation $1 + \left(\frac{dy}{dx}\right)^2 = x$ is _____ .

Q.4) If $y = Ae^{5x} + Be^{-5x}$, then $\frac{d^2y}{dx^2}$ is equal to

(a) 25 y (b) 5 y (c) −25 y (d) 15 y

Q.5) If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is

(a) $x^2 + \log|x| + C$ (b) $\frac{x^2}{2} + \log|x| + C$ (c) $\frac{x}{2} + \log|x| + C$ (d) $\frac{x}{2} - \log|x| + C$

Q.6) If the radius of the circle is increasing at the rate of 0·5 cm/s, then the rate of increase of its circumference is _____ .

Q.7) $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$ is equal to

(A) $\tan(xe^x) + c$

(B) $\cot(xe^x) + c$

(C) $\cot(e^x) + c$

(D) $\tan[e^x(1+x)] + c$

Q.8) The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is

(A) 0

(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\sqrt{3}$

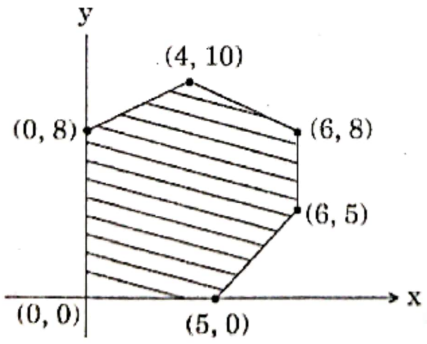
Q. 9) The vector equation of XY-plane is

(A) $\vec{r} \cdot \hat{k} = 0$

(B) $\vec{r} \cdot \hat{j} = 0$

(C) $\vec{r} \cdot \hat{i} = 0$

Q.10) The feasible region for an LPP is shown below :
Let $z = 3x - 4y$ be the objective function. Minimum of z occurs at



(A) (0, 0)

(B) (0, 8)

(C) (5, 0)

(D) (4, 10)

SECTION B

Q.11) Check if the relation R on the set A = { 1, 2, 3, 4, 5, 6 } defined as R = { (x, y) : y is divisible by x } is (i) symmetric (ii) transitive

Q.12) Find the value of $sin^{-1}[sin\left(\frac{13\pi}{7}\right)]$

Q.13) Show that the function f defined by $f(x) = (x - 1)\, e^x + 1$ is an increasing function for all $x > 0$.

Q.14) Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

Q.15) Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

Q.16) Evaluate the integral $\int(2x + 3)\, dx$

Q.17) Evaluate $\int_0^{2\pi} |\sin x|\, dx$

Q. 18) Compute P (A ∩ B), Where P (A)=0.8, P(B)= 0.5 and P(A/B) =0.4

Q. 19) If P(A) =0.25 then find P(not A)

Q.20) If A is a symmetric Matric, then show that A-A' is a skew symmetric matrix.

SECTION C

Q.21) Solve the differential equation: $ydx + (x - y^2)dy = 0$

Q.22) Evaluate: $\int_0^4 |x - 1|\, dx$

Q.23) If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$.

Q.24) **Find the shortest distance between the two lines:**

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$$

and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

Q.25) If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ *and* $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ are orthogonal.

Q.26) Solve the following Linear Programming Problem graphically:

Maximize Z = 400x + 300y subject to $x + y \leq 200, x \leq 40, x \geq 20, y \geq 0$

Q.27) Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$, $\forall x \in R$ is neither one-one nor onto.

Q.28) The probability distribution of a random variable X, where k is a constant is given below :

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx^2, & \text{if } x = 1 \\ kx, & \text{if } x = 2 \text{ or } 3 \\ 0, & \text{otherwise} \end{cases}$$

Determine

(a) the value of k

(b) $P(x \leq 2)$

SECTION D

Q.29) Solve the following system of equations by matrix method :

$$x - y + 2z = 7$$
$$2x - y + 3z = 12$$
$$3x + 2y - z = 5$$

OR

If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that $A^3 - 4A^2 - 3A + 11\,I = O$. Hence find A^{-1}.

Q. 30) Evaluate: $\int \frac{dx}{\sqrt{5 - 4x - x^2}}$

OR

Find $\int \frac{(x^3+x+1)}{(x^2-1)}\,dx$

Q. 31) If $y = (\tan^{-1} x)^2$ **, show that** $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

OR

Find $\frac{dy}{dx}$, if $x^y \cdot y^x = x^x$.